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THE CONSOLIDATION OF A FINITE LAYER SUBJECT TO SURFACE LOADING

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Abstract—In this paper a solution to the problem of the consolidation of a clay layer resting on a rough rigid base and subject to general surface loading is given.

The solution is evaluated for the particular cases of a uniformly loaded strip, circle and square for a variety of Poisson's ratio's. The results are compared with previous solutions which have used the less realistic assumption that the clay layer rests on a smooth rigid base.

1. INTRODUCTION

In investigating the consolidation of saturated clay it is usual to assume that: the soil consists of an isotropic perfectly elastic skeleton saturated with water, it is then assumed that strain of the medium is related to the effective stress by Hooke's Law and that the flow of water through the soil skeleton is governed by Darcy's Law. The equation governing one-dimensional consolidation process was developed by Terzaghi[1]. Biot[2] extended the theory to include the more general three-dimensional situation and subsequently[3–5] extended his analysis to include the effects of anisotropy and visco-elasticity.

Most solutions of Biot's equations have dealt with the surface loading of a half space[6–9] and are appropriate to the settlement of a foundation resting on a thick clay layer. More recently Gibson *et al.*[10] have obtained the solution to the problem of a uniform clay layer resting on a smooth rigid base subject to circular or strip loading. In this paper the consolidation of a uniform strip subject to a general normal surface loading is considered with the more realistic assumption that the lower surface of the strip adheres completely to a rigid base.

The approach adopted is in many respects similar to that of Gibson *et al.*[10], it differs however in one important respect in that the properties of the Laplace transformation (analyticity, nature of poles, etc.) are deduced from general theorems and the governing equations, rather than from explicit formulae. This has the effect of considerably reducing the labour in finding a solution and of making the approach rather more general than that given in Ref.[10].

2. BASIC EQUATIONS

The notation is substantially that of Gibson and NcNamee[9]. Ox, Oy, Oz are cartesian axes, the displacement vector $\mathbf{u}$ has components u, v, w; the excess pore pressure is denoted by σ and the stress tensor has components $\sigma_{xx}, \sigma_{xy}, \sigma_{xz}, \ldots$ (compression is taken as positive).

The geometry of the problem is shown schematically in Fig. 1. The upper surface is

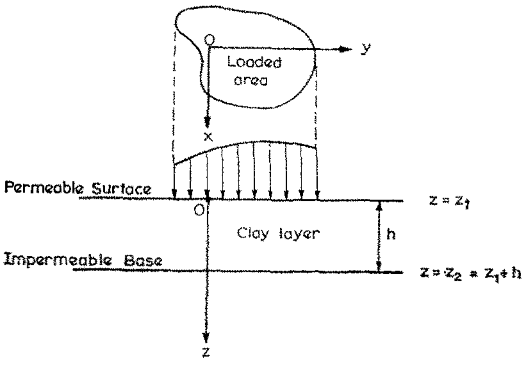


Fig. 1.

assumed to be permeable and is subject to prescribed normal tractions:

$$\sigma_{zz} = q(x, y) \qquad \text{when } z = z_1 \tag{1a}$$

$$\sigma_{xz} = 0 \qquad \text{when } z = z_1 \tag{1b}$$

$$\sigma_{yz} = 0 \qquad \text{when } z = z_1 \tag{1c}$$

$$\sigma \;\; = 0 \qquad \text{when } z = z_1. \tag{1d}$$

The lower surface completely adheres to the rigid base and will be assumed to be impermeable

$$(u, v, w) = 0 \qquad \text{when } z = z_2 \tag{2a}$$

$$\frac{\partial \sigma}{\partial z} = 0 \qquad \text{when } z = z_2. \tag{2b}$$

The equations governing the three-dimensional consolidation process have been deduced by Biot[2], in terms of the displacements $\mathbf{u}$, and the excess pore pressure σ they may be written:

$$G\nabla^2\mathbf{u} - (\lambda + G)\nabla e_v = \nabla\sigma \tag{3}$$

$$\frac{k}{\gamma_w}\nabla^2\sigma = \frac{\partial e_v}{\partial t} \tag{4}$$

where

$$e_v = -\nabla . \mathbf{u}$$

and where λ, G are Lamé's parameters for the soil skeleton, k is its coefficient of permeability and γ_w is the density of water.

The soil strains are related to the effective stresses through Hooke's law and thus typically:

$$\sigma_{zz} = \sigma + \lambda e_v - 2G\frac{\partial w}{\partial z} \tag{5a}$$

$$\sigma_{xz} = -G\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right). \tag{5b}$$

The pore water is assumed to be incompressible, it therefore follows that initially when a load is applied there can be no volume change and thus

$$e_v = 0 \qquad \text{when } t = 0+. \tag{6}$$

The required solution is the solution of equations (3, 4), which satisfies the boundary conditions (1, 2) and the initial conditions (6).

It is possible to deduce the initial and final responses. For large times it is clear that all excess pore pressures will have dissipated and thus as $t \to \infty$ the solution will reduce to that for a purely elastic material with elastic constants λ, G. This final solution will be denoted by the subscript f and thus

$$(u, v, w, \sigma) = (u_f, v_f, w_f, 0) \qquad \text{as } t \to \infty. \tag{7}$$

For small times the material is incompressible and it can be shown[11] that the displacements are identical with those of an incompressible elastic material with shear modulus G. This initial solution will be denoted by the subscript i.

$$(\mathbf{u}, \sigma) = (\mathbf{u}_i, \sigma_i) \tag{8}$$

where $\sigma_i = \frac{1}{3}(\sigma_{xxi} + \sigma_{yyi} + \sigma_{zzi})$.

3. SOLUTION OF EQUATIONS

Equations (3, 4) are mostly easily solved by applying a double Fourier transform, followed by a Laplace transform:

$$\begin{bmatrix} \mathbf{u}^F \\ \sigma^F \end{bmatrix} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathrm{e}^{-(i\alpha x + i\beta y)} \begin{bmatrix} \mathbf{u} \\ \sigma \end{bmatrix} \mathrm{d}x\, \mathrm{d}y \tag{9}$$

$$\begin{bmatrix} \mathbf{u}^{LF} \\ \sigma^{LF} \end{bmatrix} = \int_0^{\infty} \mathrm{e}^{-st} \begin{bmatrix} \mathbf{u} \\ \sigma \end{bmatrix} \mathrm{d}t. \tag{10}$$

The solution of equations (3, 4) subject to the initial condition (6) may be then written in the form

$$\begin{bmatrix} u^{LF} \\ \dfrac{\partial \sigma^{LF}}{\partial z} \end{bmatrix} = A\mathbf{p} \tag{11}$$

where A is the 4×8 matrix given in Table 1 and $\mathbf{p}T = (p_1, \ldots, p_8)$ is a vector of eight constants which must be determined from the transformed version of equations (1, 2).

Equation (12) may be used to derive other quantities of interest, in particular

$$\begin{bmatrix} \sigma^{LF} \\ \sigma_{xz}^{LF} \\ \sigma_{yz}^{LF} \\ \sigma_{zz}^{LF} \end{bmatrix} = B\mathbf{p} \tag{12}$$

where B is a 4×8 matrix given in Table 2. The quantities of $\mathbf{p}$ may now be determined from the boundary conditions (1, 2), they satisfy the equations

$$C\mathbf{p} = q^F \mathbf{e}/s \tag{13}$$

where $\mathbf{e}^T = (0, 0, 0, 0, 0, 0, 0, 1)$ and C is the partioned matrix

$$C = \begin{bmatrix} A(z_1) \\ \hline B(z_2) \end{bmatrix}.$$

Table 1

$A =$

$\frac{i\alpha c}{s} e^{\delta z}$	$\frac{i\alpha c}{s} e^{-\delta z}$	$i\alpha z\, e^{\gamma z}$	$i\alpha z\, e^{-\gamma z}$	$e^{\gamma z}$	0	$e^{-\gamma z}$	0
$\frac{i\beta c}{s} e^{\delta z}$	$\frac{i\beta c}{s} e^{-\delta z}$	$i\beta z\, e^{\gamma z}$	$i\beta z\, e^{-\gamma z}$	0	$e^{\gamma z}$	0	$e^{-\gamma z}$
$\frac{i\delta c}{s} e^{\delta z}$	$-\frac{i\delta c}{s} e^{-\delta z}$	$+(\gamma z - 1)\, e^{\gamma z}$	$-(\gamma z + 1) e^{-\gamma z}$	$-\frac{i\alpha}{\gamma} e^{\gamma z}$	$-\frac{i\beta}{\gamma} e^{\gamma z}$	$+\frac{i\alpha}{\gamma} e^{-\gamma z}$	$+\frac{i\beta}{\gamma} e^{-\gamma z}$
$+(\lambda + 2G)\delta e^{\delta z}$	$-(\lambda + 2G)\delta e^{\delta z}$	$2G\gamma^2 e^{\gamma z}$	$2G\gamma^2 e^{-\gamma z}$	0	0	0	0

$\delta = \left(\alpha^2 + \beta^2 + \frac{s}{c}\right)^{1/2}$ For definiteness the square root is assumed to take its principal value.

$\gamma = (\alpha^2 + \beta^2)^{1/2}$ For definiteness the square root is assumed to take its principal value.

$c = \frac{k}{\gamma_w}(\lambda + 2G)$ Coefficient of consolidation.

Table 2

$B =$

$(\lambda + 2G)e^{\delta z}$	$(\lambda + 2G)e^{-\delta z}$	$+2G\gamma e^{\gamma z}$	$-2Ge^{-\gamma z}$	0	0	0	0
$-2Gi\frac{\alpha\delta c}{s} e^{\delta z}$	$+2Gi\frac{\alpha\delta c}{s} e^{-\delta z}$	$-2Gi\alpha\gamma z\, e^{\gamma z}$	$+2Gi\alpha\gamma z\, e^{-\gamma z}$	$-\frac{G(\gamma^2 + \alpha^2)}{\gamma} e^{\gamma z}$	$-\frac{G\alpha\beta}{\gamma} e^{\gamma z}$	$\frac{+G(\gamma^2 + \alpha^2)}{\gamma} e^{-\gamma z}$	$+\frac{G\alpha\beta}{\gamma} e^{\gamma z}$
$2Gi\frac{\beta\delta c}{s} e^{\delta z}$	$+2Gi\frac{\beta\delta c}{s} e^{-\delta z}$	$-2Gi\beta\gamma z e^{\gamma z}$	$-2Gi\beta\gamma z e^{-\gamma z}$	$-\frac{G\alpha\beta}{\gamma} e^{\gamma z}$	$\frac{-G(\gamma^2 + \beta^2)}{\gamma} e^{\gamma z}$	$+\frac{G\alpha\beta}{\gamma} e^{-\gamma z}$	$+\frac{G(\alpha^2 + \beta^2)}{\gamma} e^{-\gamma z}$
$2G\left(1 - \frac{\delta^2 c}{s}\right) e^{\delta z}$	$2G\left(1 - \frac{\delta^2 c}{s}\right) e^{-\delta z}$	$2G\gamma(1 - \gamma z)e^{\gamma z}$	$2G\gamma(1 + \gamma z)e^{-\gamma z}$	$2Gi\alpha e^{\gamma z}$	$2Gi\beta e^{\gamma z}$	$2Gi\alpha e^{-\gamma z}$	$2Gi\beta e^{-\gamma z}$

The problem is now solved in s, α, β, z space it remains to return to the physical domain.

The first problem is to invert the Laplace transform, for definiteness let us focus our attention on one variable, say w, then

$$w^{LF} = \mathbf{a}_4^T\mathbf{p} \tag{14}$$

where $\mathbf{a}_4^T$ is the fourth row of the matrix A given in Table 1. We shall now attempt to expand w^{LF} in terms of its poles

$$w^{LF} = \frac{\rho_0(\alpha, \beta, z)}{s} + \sum_{n=1}^{\infty} \frac{\rho_n(\alpha, \beta, z)}{s - s_n(\alpha, \beta)}. \tag{15}$$

A set of necessary conditions for this expansion is given by Copson[12].

First notice that w^{LF}, as defined by equations (13, 14) is an analytic function of s save possibly for a branch line $L - \infty < s/c < -\gamma^2$. The function w^{LF} will therefore be analytic provided it is continuous across L. In order to establish this consider a point p_1, $s = s_0 + 0i$ and denote values at this point by the subscript 1, then:

$$C(\delta_1)\mathbf{p}_1 = q^F\mathbf{e}/s_0$$
$$w_1 = \mathbf{a}^T(\delta_1, z)\mathbf{p}_1.$$

Likewise considering a point P_2, $s = s_0 - 0i$ and adopting a similar notation:

$$C(\delta_2)\mathbf{p}_2 = q^F\mathbf{e}/s_0$$
$$w_2 = a^T(\delta_2, z)\mathbf{p}_2.$$

Now notice that:

$$\delta_2 = -\delta_1$$
$$C(\delta_2) = C(\delta_1)H$$
$$a^T(\delta_2) = a^T(\delta_1)H$$

where H is the permutation matrix obtained by interchanging the first and second columns of an 8×8 unit matrix.

The above equations imply that:

$$\mathbf{p}_2 = H^{-1}\mathbf{p}_1$$

and thus that

$$w_2^{LF} = \mathbf{a}^T(\delta_1, z)HH^{-1}\mathbf{p}_1 = w_1^{LF}$$

so that w^{LF} is analytic throughout the entire s plane.

Next it will be shown that $w^{LF} \to 0$ when $s \to \infty$. This is most easily demonstrated by an application of the Tauberian theorems for Laplace transforms, Van der Pol and Bremmer [13]. In their simplest form these state that if $y(t) \to y_0$ when $t \to 0+$ then $sy^L(s) \to y_0$ as $s \to \infty$.

Now as was mentioned previously

$$w \to w_i \quad \text{when} \quad t \to 0+$$

and thus

$$\lim_{s \to \infty} sw^{LF} = w_i^F$$

or alternatively

$$w^{LF} \rightarrow \frac{w_\iota^F}{s} \quad \text{when} \quad s \rightarrow \infty. \tag{16}$$

It now only remains to verify that the conditions regarding the distribution of poles are met.

It is clear from equations (13, 14) that w^{LF} has a pole at $s = 0$, while its remaining poles are given by the zeros of:

$$\Delta = \text{Det } C. \tag{17}$$

It can be shown that these are all negative (see Appendix A). In general these zeros must be found numerically and it can be verified that the necessary conditions on the distribution of poles, for validity of the expansion (15), are met.

The labour involved in finding these roots may be considerably reduced when it is noted that the value of Δ and thus the values of its zeros s_n depend only on the value of γ not jointly an α, β. That is to say $s_n = s_n(\alpha, \beta) = s_n((\alpha^2 + \beta^2)^{1/2}) = s_n(\gamma)$. A simple method of obtaining this result is to set $\alpha = \gamma \cos \psi$ $\beta = \gamma \sin \psi$ and to premultiply, and post multiply the matrix C by the orthogonal matrices H_b, H_a respectively, where

$$H_b = \begin{bmatrix} R_\psi & 0 & 0 & 0 \\ 0 & I_3 & 0 & 0 \\ 0 & 0 & R_\psi & 0 \\ 0 & 0 & 0 & I_1 \end{bmatrix}, \qquad H_a = \begin{bmatrix} I_4 & 0 & 0 \\ 0 & R_\psi^T & 0 \\ & 0 & R_\psi^F \end{bmatrix} \tag{18}$$

R_ψ denotes the planar rotation

$$R_\psi = \begin{bmatrix} \cos \psi, & \sin \psi \\ -\sin \psi & \cos \psi \end{bmatrix}$$

and I_n denotes unit matrix of rank n.

It follows quite directly that the resultant transformed matrix depends only on γ, and so the desired result is obtained.

This of course implies that the poles s_n are the same for the general case, and the cases of plane strain and circular symmetry, there is a distinct computational advantage in calculating s_n using the plane strain formulation, i.e. from the matrix obtained by setting $\alpha = \gamma$, $\beta = 0$ and omitting the 6th and 8th columns and the 2nd and 7th rows of C.

A more elaborate analysis of Δ shows that its zeros have the asymptotic form

$$\delta_n = (s_n/c + \gamma^2)^{1/2} \rightarrow \varepsilon(\gamma) + n\pi/(z_2 - z_1)$$

where $\varepsilon(\gamma)$ is independent of n but depends on γ.

The conditions for validity of the expansion have now all been satisfied, it remains to determine the residues $\rho_0, \rho_1, \ldots,$ Consider first the pole at $s = 0$, it follows in quite straight forward fashion that:

$$\rho_0 = \mathbf{a}_4^T \mathbf{p}_0$$

where $\mathbf{p}_0$ is determined from the equations

$$C\mathbf{p}_0 = q^F \mathbf{e}$$

and it is understood that all quantities are evaluated at $s = 0$.

It may also be shown, either by use of equation (7), or by means of the Tauberian theorems for Laplace transforms that

$$\rho_0 = w_f^F. \tag{19}$$

Next consider a pole at $s = s_n(\alpha, \beta)$, it may be verified that

$$\rho_n = \frac{q^F(\mathbf{l}_n^T\mathbf{e})(\mathbf{a}_4^T\mathbf{r}_n)}{s(\mathbf{l}_n^T C'\mathbf{r}_n)}$$

where $\mathbf{r}_n$, $\mathbf{l}_n$ are solutions of the equations

$$C\mathbf{r}_n = 0, \qquad \mathbf{l}_n^T C = 0$$

and $C' = \partial C/\partial s$ and it is understood that all quantities are evaluated at $s = s_n(\alpha, \beta)$.

The quantities in the expansion (15) have all now been evaluated,† the Laplace transform may be inverted to give,

$$w^F = \rho_0 + \sum_{n=1}^{\infty} \rho_n e^{s_n(\alpha,\beta)t} \tag{20}$$

then the double Fourier transform may be inverted so that

$$w = \tfrac{1}{4}\pi^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[\rho_0 + \sum_{n=1}^{\infty} \rho_n e^{s_n(\alpha,\beta)t}\right] e^{i(\alpha x+\beta y)}\, d\alpha\, d\beta$$

whereupon using equation (17) it is found that:

$$w = w_f + \tfrac{1}{4}\pi^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[\sum_{n=1}^{\infty} \rho_n e^{s_n(\alpha,\beta)t}\right] e^{(i\alpha x+i\beta y)}\, d\alpha\, d\beta. \tag{21}$$

This expression may now be evaluated numerically. A convenient check on the accuracy of the calculation is provided by setting $t = 0$, then

$$w_i = w_f + \tfrac{1}{4}\pi^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left(\sum_{n=1}^{\infty} \rho_n\right) e^{i(\alpha x+\beta y)}\, d\alpha\, d\beta$$

which may be checked against known elastic values (15).

4. RESULTS

The expressions developed in the previous section have been evaluated for the following three cases ‡.

(a) *A uniformly loaded strip*

$$\left.\begin{array}{ll}\sigma_{zz} = q & -a \le x \le a\\ \sigma_{zz} = 0 & \text{elsewhere}\end{array}\right] z = z_1.$$

In this case

$$q^F = q\frac{\sin \alpha a}{\alpha}\,\delta(\beta)$$

where $\delta(\beta)$ is the Dirac delta function.

The effect of this delta function is to reduce the order of equations (13) from eight to six and to reduce the inversion of the double Fourier transform (5) to a single Fourier transform.

† It was found convenient to verify the validity of this expansion for particular values of s, this had the effect of providing a check on the accuracy of numerical calculations and ensured that no poles $s = s_n(\alpha, \beta)$ had been omitted from the expansion.

‡ The solutions for a uniformly loaded rectangle have also been calculated and these will appear in a subsequent publication.

(b) *A uniformly loaded circle*

$$\left.\begin{array}{ll}\sigma_{zz} = q & 0 < r < a \\ \sigma_{zz} = 0 & \text{elsewhere}\end{array}\right] z = z_1.$$

As has been pointed out previously[6, 7, 10] there is a strong similarity between the problem of plane strain and circular symmetry. The similarity stems from equation (18) and from the well known fact, that in the case of problems exhibiting circular symmetry, Poisson's integral can be used to convert a double Fourier transform to a Hankel transform of order zero, Sneddon[16]. Thus:

$$q^F = 2\pi q \frac{a}{\gamma} J_1(\gamma a)$$

and as stated above the double Fourier inversion in equation (20) can be replaced by a single Hankel transform of order zero.

(c) *A uniformly loaded square*

$$\left.\begin{array}{ll}\sigma_{zz} = q & -a \le x \le a \\ & -a \le y \le a \\ \sigma_{zz} = 0 & \text{elsewhere}\end{array}\right] z = z_1.$$

Thus

$$q^F = 4q \frac{\sin \alpha a}{\alpha} \frac{\sin \beta a}{\beta}.$$

The results of these three cases have been given for a range of values of h/a (0·2, 0·5, 1·0, 2·0, 5·0) and Poisson's ratio $\nu = 0{\cdot}0, 0{\cdot}25, 0{\cdot}48$ (Figs. 2–7). For the most part they are plotted in the form of a degree of settlement.

$$U = \frac{w - w_i}{w_f - w_i}$$

where w denotes the deflection at the centre of the footing and the subscripts i and f denote initial and final values.

Particular cases of the central deflections are compared with those for a smooth base in Fig. 3 and these results show that the nature of the base constraint (rough or smooth) may have a significant effect on the time settlement behaviour. It is interesting to note that if instead of actual deflections, degrees of settlement are compared the effect is reduced considerably.

It was found that, as has been suggested by Davis and Poulos[17], the settlement of a square is in practice very little different from that of a circle of equal area, the major departure from this approximation occurs in the range $0{\cdot}5 < h/a < 1{\cdot}0$ and a typical case is shown in Fig. 7.

Referring to Figs. 4–6 it can be seen that the rate of settlement decreases as a/h increases, that is to say, for a layer of fixed depth h, an increase in width of the footing will lead to a reduction in the rate of settlement.

The results for the rate of settlement of a strip footing have been compared with a finite element solution and a diffusion solution in[18], this comparison shows that the rate of settlement calculated by the diffusion method compares quite favourably with the exact solution and indicates the approach elaborated in[17] may be used with some confidence.

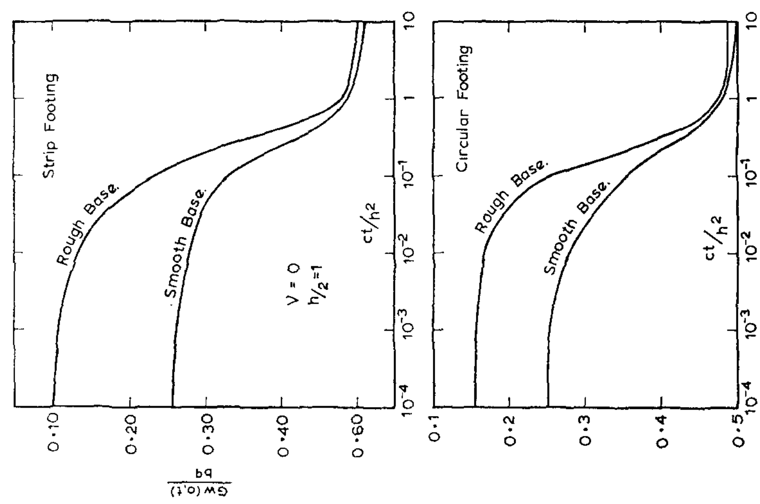

Strip Footing
Rough Base.
Smooth Base.
ν = 0
h/2=1
ct/h²
Gw(0,t)/bq
Circular Footing
Rough Base.
Smooth Base.
ct/h²

Fig. 2.

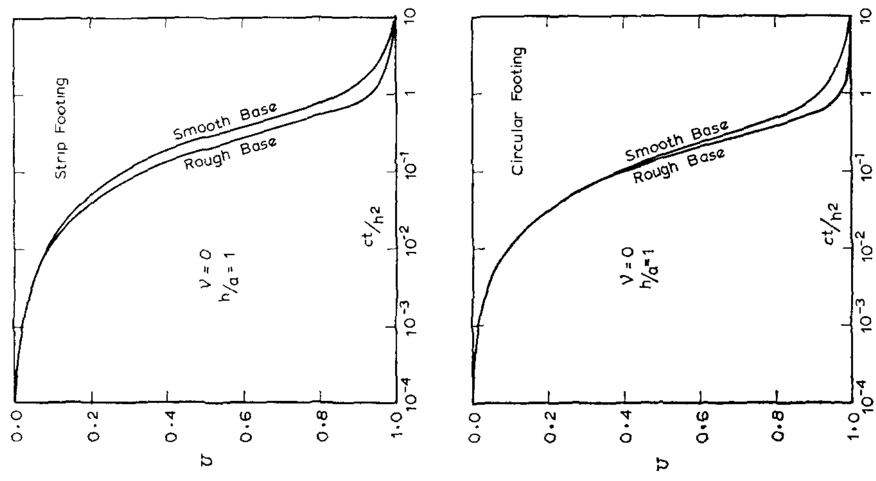
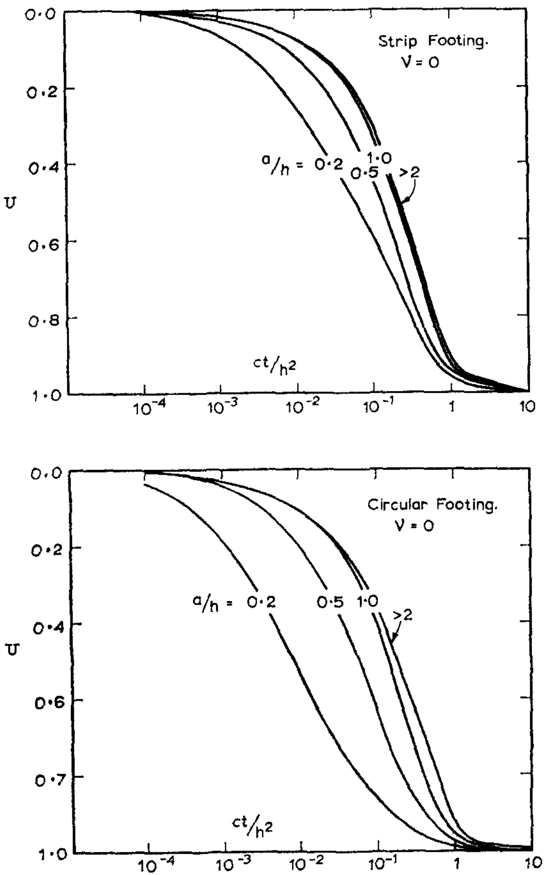
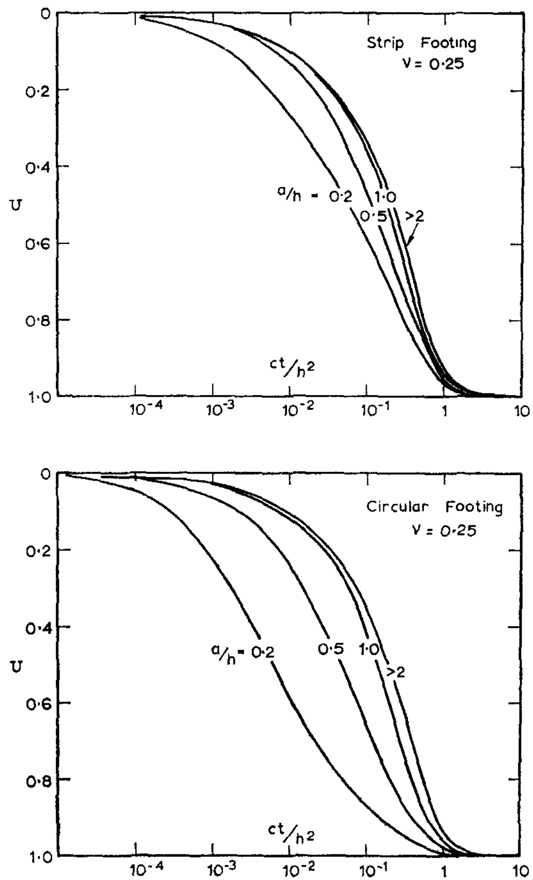

Strip Footing
Smooth Base
Rough Base
ν = 0
h/a = 1
ct/h²
U
Circular Footing
Smooth Base
Rough Base
ν = 0
h/a=1
ct/h²
U

Fig. 3.


Strip Footing.
V = 0
a/h = 0·2 0·5 1·0 >2
U
ct/h2
0·0
0·2
0·4
0·6
0·8
1·0
10-4
10-3
10-2
10-1
1
10
Circular Footing.
V = 0
a/h = 0·2 0·5 1·0 >2
U
ct/h2
0·0
0·2
0·4
0·6
0·7
1·0
10-4
10-3
10-2
10-1
1
10
Fig. 5.


Strip Footing
V = 0·25
a/h = 0·2 0·5 1·0 >2
U
ct/h2
0
0·2
0·4
0·6
0·8
1·0
10-4
10-3
10-2
10-1
1
10
Circular Footing
V = 0·25
a/h = 0·2 0·5 1·0 >2
U
ct/h2
0
0·2
0·4
0·6
0·8
1·0
10-4
10-3
10-2
10-1
1
10
Fig. 4.

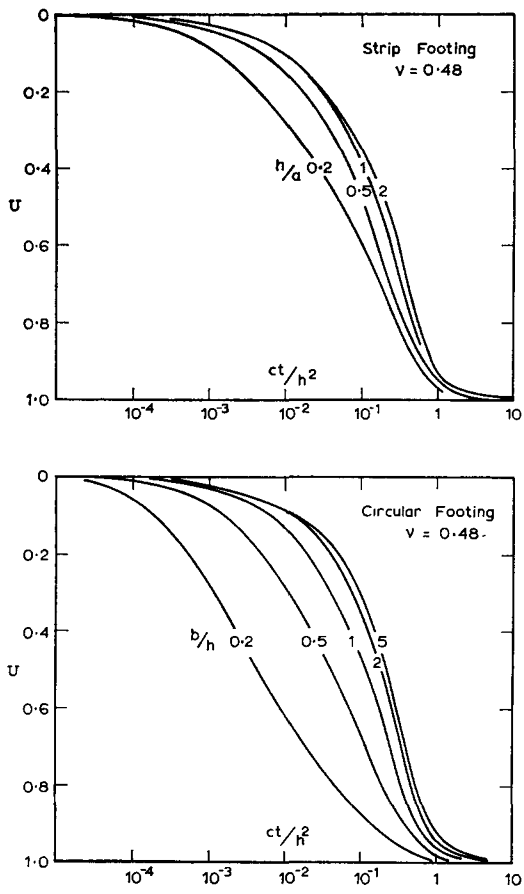

Strip Footing
ν = 0·48
h/a 0·2
0·5
1
2
U
ct/h²
Circular Footing
ν = 0·48
b/h 0·2
0·5
1
2
5

Fig. 6.

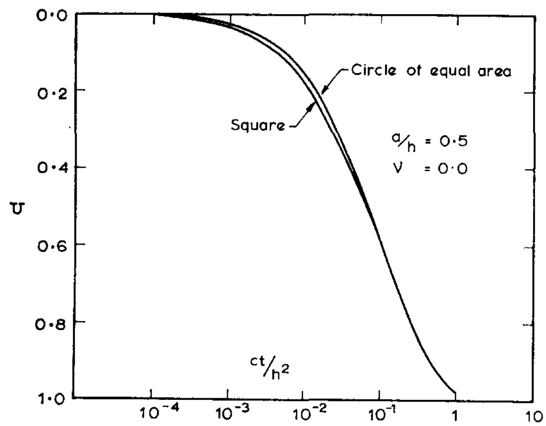

Circle of equal area
Square
a/h = 0·5
ν = 0·0
U
ct/h²

Fig. 7.

5. CONCLUSIONS

In this paper a solution to the problem of the consolidation of a clay layer resting on a rough rigid base and subject to general surface loading is given.

The solution is evaluated for the particular cases of a uniformly loaded strip, circle and square for a variety of Poisson's ratios. The results are compared with previous solutions which have used the less realistic assumption that the clay layer rests on a smooth rigid base. It is found that if actual settlements are compared there is a marked difference between the rough and smooth base but if the degree of settlement is considered then this effect is considerably reduced.

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Абстракт — В работе дается решение задачи консолидации слоя глины, лежащего на шереховатым жестком основании и подверженного действию общей поверхности нагрузки. Определяется решение для частных случаев равномерно нагруженной полосы, круга и квадрата, для разных коэффициентов Пуассона. Сравниваются результаты с предидущими решениями, которые пользовались менее действительным предположением, в котором слой глины операется на гладком, жестком основании.

APPENDIX A

In this Appendix it will be shown that the poles $s = s_n(\alpha, \beta)$ which occur in expansion (15) all lie on the negative real axis. Although such a result would be suspected on physical grounds the proof is not altogether obvious.

In order to provide such a proof consider the set of displacements, pore pressures, stresses defined by the equations:

$$\begin{aligned}
\mathbf{u} &= u^F(\alpha, \beta, z)e^{-i(\alpha x+\beta y)}\\
\sigma &= \sigma^F(\alpha, \beta, z)e^{-i(\alpha x+\beta y)}\\
\sigma_{zz} &= \sigma_{zz}^F(\alpha, \beta, z)e^{-i(\alpha x+\beta y)}
\end{aligned}$$

in a finite region V bounded by the six planes $z = z_1$, $z = z_2$, $x = \pm\pi/\alpha$, $y = \pm\pi/\beta$. These quantities may be found by superimposing the solution of the four consolidation problems defined by the boundary conditions shown in Table 1.

Each of these set of boundary conditions poses a consolidation problem of the type dealt with in[14] and thus the poles of the solutions to each of these problems all lie on the negative real axis. The general solution required in this paper may be found by superimposing these solutions as follows:

$$\text{Solution} = q^F(\alpha, \beta) \times [(\text{Solution}_{\text{I}} - \text{Solution}_{\text{IV}}) + i(\text{Solution}_{\text{II}} + \text{Solution}_{\text{III}})].$$

Thus providing the required result.